Univ.-Prof. Dr. Matthias Ehrhardt
Dipl.-Math. Alexander Schranner
WiSe 2009/10

Bergische Universität
Wuppertal
Angewandte Mathematik
Numerische Analysis

Introduction to Numerical Methods for Computer Simulation

Exercises No. 3

E 9: *Tridiagonal Matrices*

Given the tridiagonal matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & & & 0 \\ a_{21} & a_{22} & a_{23} & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & a_{n-1,n} \\ 0 & & & a_{n,n-1} & a_{n,n} \end{pmatrix}.$$

a) First, we compute the triangular decomposition $A = L \cdot U$ without pivoting.
(Assumption: each pivot is nonzero).
Write the algorithm for this special case.
(What changes arise in comparison to a general matrix $A \in \mathbb{R}^{n \times n}$?)
How do L and U look like?
What is the complexity of the LU-decomposition here?

b) If we use partial pivoting, we obtain the decomposition $P \cdot A = L \cdot U$ with permutation matrix P.
Do L and U have the same structure as in a)?

c) During LU-decomposition of a general matrix A, the computed entries u_{ij} in the upper triangular matrix $U = (u_{ij})$ may grow (compared to A). For tridiagonal matrices and using partial pivoting, prove the estimate

$$|u_{ij}| \leq 2 \cdot p_0 \,, \qquad p_0 := \max_{1 \leq i,j \leq n} |a_{ij}| \,.$$

E 10: *LU-Decomposition of Block Matrix*

Given the matrix

$$A = \begin{pmatrix} \tilde{A} & v \\ w^\top & \alpha \end{pmatrix} \in \mathbb{R}^{(n+1)\times(n+1)},$$

where $\tilde{A} \in \mathbb{R}^{n \times n}$ is regular, $v, w \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$.

a) Determine the LU-decomposition of A for given $\tilde{A} = \tilde{L} \cdot \tilde{U}$.

b) Show: A is regular $\iff \alpha - w^\top \tilde{A}^{-1} v \neq 0$.