Univ.-Prof. Dr. Matthias Ehrhardt
Dipl.-Math. Alexander Schranner
WiSe 2009/10

Bergische Universität
Wuppertal
Angewandte Mathematik
Numerische Analysis

Introduction to Numerical Methods for Computer Simulation

Exercises No. 4

E 11: *LU- and Cholesky-Decomposition*

We consider the symmetric tridiagonal matrix:

$$A = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

a) Compute the LU-decomposition of A.

b) Proof: A is positive definite.

c) How does a $LDL^\top$-decomposition of A look like (rational Cholesky)?

E 12: *Residual of linear systems.*

Given the linear system $Ax = b$, where

$$A = \begin{pmatrix} 0.78 & 0.563 \\ 0.913 & 0.659 \end{pmatrix}, \qquad b = \begin{pmatrix} 0.217 \\ 0.254 \end{pmatrix}.$$

a) Compute the solution x.

b) Let there be given the following two approximations to x:

$$x_1 = \begin{pmatrix} 0.999 \\ -1.001 \end{pmatrix} \quad \text{und} \quad x_2 = \begin{pmatrix} 0.341 \\ -0.087 \end{pmatrix}.$$

The residual $r(x_i)$ is defined by $r(x_i) = Ax_i - b$, $i = 1, 2$. Compute r and interprete the result.

E 13: *Sensitivity of linear systems.*

Consider the linear system $Ax = b$, where

$$A = \begin{pmatrix} 10 & 11 \\ 9 & 10 \end{pmatrix} \qquad \text{and} \qquad b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

a) Sketch the image $\Phi(\mathcal{K}) := \{\Phi(x) : x \in \mathcal{K}\}$ of the set

$$\mathcal{K} := \{x \in \mathbb{R}^2 : \|x\|_\infty = 1\}$$

with respect to the linear mapping $\Phi(x) := Ax$.

b) Compute the condition number of A with respect to the norm $\|\cdot\|_\infty$.

c) Estimate the relative error in the solution of the linear system with respect to the norm $\|\cdot\|_\infty$, if the computations are done with a perturbed matrix $\tilde{A}$ and perturbed right-hand side $\tilde{b}$, where it holds

$$\|\tilde{A} - A\|_\infty \leq 0.02 \qquad \text{and} \qquad \|\tilde{b} - b\|_\infty \leq 0.001.$$

d) Compute the exakt solution of the linear system $Ax = b$. Calculate also the exact solution of the perturbed linear system $\tilde{A}\tilde{x} = \tilde{b}$, where

$$\tilde{A} - A = \begin{pmatrix} -0.01 & 0.01 \\ 0.01 & -0.01 \end{pmatrix} \qquad \text{and} \qquad \tilde{b} - b = \begin{pmatrix} -0.001 \\ 0.001 \end{pmatrix}.$$

Compare the difference between both solutions with the estimate obtained in (c).