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Bergische Universität
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Angewandte Mathematik
Numerische Analysis

Introduction to Numerical Methods for Computer Simulation

Exercises No. 5

E 14: *Linear Least Squares Problem*

The data (t_i, y_i), $i = 1, \ldots, 7$ with

$$t_1 > t_2 > t_3 > 0\,,\ t_4 = 0\,,\ t_5 = -t_3\,,\ t_6 = -t_2\,,\ t_7 = -t_1$$

is to be fit with a quadratic ansatz function.

a) Determine the coefficient matrix A to solve the least squares problem

$$\min_x \|Ax - y\|_2^2\ .$$

b) Compute the normal equations $A^\top Ax = A^\top y$.

E 15: *Householder Transformation Matrix*

The matrix

$$T = I - 2vv^T = \begin{pmatrix} 1-2v_1^2 & -2v_1v_2 \\ -2v_1v_2 & 1-2v_2^2 \end{pmatrix}$$

is given for a vector $v \in \mathbb{R}^2$ with $\|v\|_2 = 1$.

a) Compute T^2. What about the inverse T^{-1}?

b) Determine $y = Tx$ for the vector $x = s + p$, where $s \perp v$ and $p \parallel v$.
What kind of geometrical mapping describes T? Draw a sketch.

c) Compute v, such that T maps the vector $x = \begin{pmatrix} -3/5 \\ -4/5 \end{pmatrix}$ to the unit vector $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.
Is it possible to find such v for an arbitrary x?

E 16: *QR and Cholesky*

a) Considering the normal equations $A^\top Ax = A^\top b$ to solve a least squares problem, the matrix $A^\top A$ might be ill-conditioned, as it can be seen in the following example.

Compute the condition number $\text{cond}_{\|\cdot\|_1}(A^\top A)$ for the special matrix

$$A = \begin{pmatrix} 1 & 1 \\ 0.1 & 0 \\ 0 & 0.1 \end{pmatrix} .$$

b) Let $A \in \mathbb{R}^{m \times n}$ with $m \geq n$. Using the QR-decomposition, a Cholesky-decomposition of $A^\top A$ can be determined directly (without computing $A^\top A$).
How can you derive a Cholesky-decomposition $A^\top A = LL^\top$ applying the QR-decomposition $A = QR$, $R = \begin{pmatrix} R' \\ 0 \end{pmatrix}$, $R' \in \mathbb{R}^{n \times n}$?