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Bergische Universität
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Numerische Analysis

Introduction to Numerical Methods for Computer Simulation

Exercises No. 6

E 17: *Classical methods.*

Consider the linear system $Ax = b$ with

$$A = \begin{pmatrix} 6 & -3 \\ -1 & 2 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 3 \\ 1 \end{pmatrix}.$$

a) Starting in $x^0 = (-4, -3)^T$, compute the first two steps of the Jacobi method as well as of the Gauss-Seidel method. Sketch the arising points x^0, x^1, x^2 in a coordinate system. Construct also the two straight lines, which represent the equations of the linear system. Describe the performance of both methods.

b) Sketch (without computations) the first approximation x^1 of the SOR method using $x^0 = (-4, -3)^T$ again in the cases $\omega = 0.8$ and $\omega = 1.2$. Explain the effect of the relaxation parameter.

E 18: *Richardson method.*

Given a linear system $Ax = b$, the iterative scheme

$$x^{k+1} = x^k - \omega(Ax^k - b), \quad k = 0, 1, 2, \ldots$$

is called *Richardson method.*

a) Specify the matrix $B(\omega)$ in the splitting

$$B(\omega)x^{k+1} + (A - B(\omega))x^k = b,$$

which yields Richardson's method.

b) Let the matrix A be positive definte, where $0 < \lambda_1 \leq \ldots \leq \lambda_n$ are the corresponding eigenvalues. Determine the spectral radius $\rho(I - B(\omega)^{-1}A)$ in dependence on the eigenvalues $\lambda_1, \ldots, \lambda_n$ and the relaxation parameter ω.

c) Given a positive definite matrix A, identify an optimal relaxation parameter ω_{opt} such that $\rho(I - B(\omega)^{-1}A)$ is minimal. Determine the complete set of parameters $\omega > 0$, where the Richardson method is convergent, i.e. $\rho(I - B(\omega)^{-1}A) < 1$ holds. Sketch the spectral radius in dependence on the relaxation parameter. Describe $\rho(I - B(\omega)^{-1}A)$ with respect to the condition number $\kappa_2(A)$.