Bergische Universität Wuppertal
Applied Mathematics and Numerical Analysis
Univ.-Prof. Dr. M. Ehrhardt Winter Term 11/12

Exercise Sheet 1 to the Lecture Course "Computational Finance"
(The Binomial Method)

<u>Task 1</u> (Solution to the Binomial Model) (3 Points)

Derive from equations (1.5), (1.9) and $u\,d=\gamma$ for some constant γ (not necessarily $\gamma=1$ as in (1.10)) the relation

$$u=\beta+\sqrt{\beta^2-\gamma} \quad \text{for} \quad \beta:=\frac{1}{2}\big(\gamma\, e^{-r\Delta t}+e^{(r+\sigma^2)\Delta t}\big).$$

<u>Task 2</u> (Anchoring the Binomial Grid at K) (4 Points (2+2))

The equation (1.10) has established a kind of symmetry for the grid. As an alternative, one may anchor the grid in another way by choosing (for even M)

$$S_0\, u^{M/2}\, d^{M/2}=K.$$

a) Give a geometrical interpretation.

b) Derive the relevant formula for u and d.

Hint: Use Task 1

<u>Task 3</u> (Price Evolution for the Binomial Method) (3 Points)

Recall that

$$\boxed{\begin{aligned} \beta &:= \frac{1}{2}\big(e^{-r\Delta t}+e^{(r+\sigma^2)\Delta t}\big)\\ u &= \beta+\sqrt{\beta^2-1}\\ d &= 1/u=\beta-\sqrt{\beta^2-1}\\ p &= \frac{e^r\Delta t-d}{u-d}. \end{aligned}} \tag{1.11}$$

For β from (1.11) and $u=\beta+\sqrt{\beta^2-1}$ show

$$u=\exp\big(\sigma\Delta t\big)+\mathrm{O}\big((\Delta t)^3\big)$$

<u>Programming Task 1</u> Implementing the Binomial Method (5 Points)

Design and implement an algorithm for calculating the value $V^{(M)}$ of a European option.

INPUT: r (interest rate), σ (volatility), T (time to expiration in years), K (strike price), S (price of asset), and the choices put or call.

Control the mesh size $\Delta t = T/M$ adaptively. For example, calculate V for $M = 8$ and $M = 16$ and in case of a signicant change in V use M = 32 and possibly $M = 64$.
Test examples:

a) put, European, $r = 0.06$, $\sigma = 0.3$, $T = 1$, $K = 10$, $S = 5$

b) call, otherwise as in a)

c) The mesh size control must be done carefully and has little relevance to error control. To make this evident, calculate for the test numbers a) a sequence of $V^{(M)}$ values, say for $M = 100, 101, 102, \dots, 150$, and plot the error $|V^{(M)} - 4.430465|$.

- **Return** the solutions until Monday, October 31, **before** the lectures.
- **Return** the solutions of programming task until Monday, November 8, **before** the lectures.