Bergische Universität Wuppertal
Applied Mathematics and Numerical Analysis
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Winter Term 11/12

Exercise Sheet 2 to the Lecture Course "Computational Finance"
(Stochastic Processes)

Task 1 (Calculating an Estimate of the Variance) (3 Points)

An estimate of the variance of M numbers $x_1, \dots, x_M$ is

$$s_M^2 := \frac{1}{M-1}\sum_{i=1}^{M}(x_i - \bar{x})^2, \quad \text{with} \quad \bar{x} := \frac{1}{M}\sum_{i=1}^{M} x_i$$

The alternative formula

$$s_M^2 := \frac{1}{M-1}\left(\sum_{i=1}^{M} x_i^2 - \frac{1}{M}\left(\sum_{i=1}^{M} x_i\right)^2\right)$$

can be evaluated with only one loop $i = 1, \dots, M$, but should be avoided because of the danger of *subtractive cancellation*. The following single-loop algorithm is recommended:

$$\begin{aligned} &\alpha_1 := x_1, \quad \beta_1 := 0 \\ \text{for} \quad &i = 2, \dots, M \ : \\ &\alpha_i := \alpha_{i-1} + \frac{x_i - \alpha_{i-1}}{i} \\ &\beta_i := \beta_{i-1} + \frac{(i-1)(x_i - \alpha_{i-1})^2}{i} \end{aligned}$$

a) Show $\bar{x} = \alpha_M$.

b) Show $s_M^2 = \beta_M/(M-1)$.

Task 2 (4 Points (2+2))

In Definition 1.7 the requirement (a) $W_0 = 0$ is dispensable. Then the requirement (b) reads

$$\mathrm{E}(W_t - W_0) = 0, \qquad \mathrm{E}\big((W_t - W_0)^2\big) = t.$$

Use these relations to deduce

$$\begin{aligned} &\mathrm{E}(W_t - W_s) = 0, \\ &\mathrm{Var}(W_t W_s) = \mathrm{E}\big((W_t - W_s)^2\big) = t - s. \end{aligned} \tag{1.21}$$

Hint: $(W_t - W_s)^2 = (W_t - W_0)^2 + (W_s - W_0)^2 - 2(W_t - W_0)(W_s - W_0)$

<u>Task 3</u> (3 Points)

Suppose that a random variable X_t satises $X_t \sim \mathcal{N}(0, \sigma^2)$. Use the formula for the *expected value* (first moment)

$$\mu := \mathrm{E}(X) := \int_{-\infty}^{\infty} x\, f(x)\, dx. \tag{B1.4}$$

to show

$$E(X_t^4) = 3\sigma^4.$$

- **Return** the solutions until Monday, November 8, **before** the lectures.