Bergische Universität Wuppertal
Applied Mathematics and Numerical Analysis
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Winter Term 11/12

Exercise Sheet 3 to the Lecture Course "Computational Finance"
(Implied Volatility)

Task 1 (Implied Volatility) (10 Points)

For European options we take the valuation formula of Black and Scholes of the type $V = v(S, \tau, K, r, \sigma)$, where τ denotes the time to maturity, $\tau := T - t$. The function v is defined as

$$d_1 := \frac{\log \frac{S}{K} + (r - \delta + \frac{\sigma^2}{2})(T - t)}{\sigma\sqrt{T - t}}, \tag{1a}$$

$$d_2 := d_1 - \sigma\sqrt{T - t}, \tag{1b}$$

$$V_C(S, t) = Se^{-\delta(T-t)}F(d_1) - Ke^{-r(T-t)}F(d_2), \tag{1c}$$

$$V_P(S, t) = -Se^{-\delta(T-t)}F(-d_1) + Ke^{-r(T-t)}F(-d_2), \tag{1d}$$

where δ is the continuous dividend yield and F denotes the standard normal cumulative distribution. If actual market data of the price V are known, then one of the parameters considered known so far can be viewed as unknown and fixed via the implicit equation

$$V - v(S, \tau, K, r, \sigma) = 0. \tag{*}$$

In this calibration approach the unknown parameter is calculated iteratively as solution of equation (*). Consider σ to be in the role of the unknown parameter. The volatility σ determined in this way is called *implied volatility* and is zero of $f(\sigma) := V - v(S, \tau, K, r, \sigma)$.

a) Implement the evaluation of V_C and V_P according to (1).

b) Design, implement and test an algorithm to calculate the implied volatility of a call. Use Newtons method to construct a sequence $x_k \to \sigma$. The derivative $f'(x_k)$ can be approximated by the difference quotient

$$\frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}.$$

For the resulting *secant iteration* invent a stopping criterion that requires smallness of both $|f(x_k)|$ and $|x_k - x_{k-1}|$.

c) Calculate the implied volatilities for the data

$$T - t = 0.211, \quad S_0 = 5290.36, \quad r = 0.0328$$

and the pairs K, V from Table 1 (for more data see `http://www.compfin.de`). Enter for each calculated value of σ the point (K, σ) into a figure, joining the points with straight lines. (You will notice a convex shape of the curve. This shape has lead to call this phenomenon *volatility smile*.)

Table 1 Calls on the DAX on January 4, 1999

K	6000	6200	6300	6350	6400	6600	6800
V	80.2	47.1	35.9	31.3	27.7	16.6	11.4

- **Return** the solutions until Monday, November 15, **before** the lectures.