Bergische Universität Wuppertal
Applied Mathematics and Numerical Analysis
Univ.-Prof. Dr. M. Ehrhardt

Winter Term 11/12

Exercise Sheet 8 to the Lecture Course "Computational Finance"
(Successive Overrelaxation Method and Front-Fixing Approach)

Task 1 (Gauß-Seidel as Special Case of SOR) (4 Points)

Let the $n \times n$ matrix $A = ((aij))$ additively be partitioned into $A = D - L - U$, with D diagonal matrix, L strict lower triangular matrix, U strict upper triangular matrix, $x, b \in \mathbb{R}^n$. The *Gauß-Seidel method* is defined by

$$(D - L)x^{(k)} = Ux^{(k-1)} + b$$

for k = 1, 2,. . . . Show that with

$$r_i^{(k)} := b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k)} - \sum_{j=i}^{n} a_{ij} x_j^{(k-1)}$$

and for $\omega_R = 1$ the relation

$$x_i^{(k)} = x_i^{(k-1)} + \omega_R \frac{r_i^{(k)}}{a_{ii}}$$

holds. For general $1 < \omega_R < 2$ this defines the SOR (successive overrelaxation) method.

Task 2 (Front-Fixing for American Options) (3+3+ 5 Points)

Apply the transformation

$$\zeta := \frac{S}{S_f(t)}, \qquad y(\zeta, t) := V(S, t)$$

to the Black-Scholes equation

$$\boxed{\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - \delta) S \frac{\partial V}{\partial S} - rV = 0.} \qquad \text{(BS)}$$

a) Show

$$\frac{\partial y}{\partial t} + \frac{\sigma^2}{2}\zeta^2 \frac{\partial^2 y}{\partial \zeta^2} + \left[(r - \delta) - \frac{1}{S_f}\frac{\mathrm{d}S_f}{\mathrm{d}t}\right]\zeta \frac{\partial y}{\partial \zeta} = 0$$

b) Set up the domain for (ζ, t) and formulate the boundary conditions for an American call. (Assume $\delta > 0$.)

c) **(Programming task)** Set up a finite-difference scheme to solve the derived boundary value problem. The curve $S_f(t)$ is implicitly defined by the above PDE, with final value $S_f(T) = \max(K, rK)$.

- **Return** the solutions until Monday, January 9, **before** the lectures.
- **Return** the solutions of programming task until Monday, January 16, **before** the lectures.