



Numerical Analysis and Simulation II: Partial Differential Equations (PDEs)

Exercise Sheet 1- Classification and Transformation of Variables

Return of Exercise Sheet: April 24, 2012 (before the lecture)

Homework 1: *Classification*

(4 Points)

1. Classify the following partial differential equations

$$u_{xx} + 2u_{xy} + 2u_{yy} + 4u_{yz} + 5u_{zz} + u_x + u_y = 0$$
$$e^z u_{xy} - u_{xx} = \log(x^2 + y^2 + z^2)$$

according to the conventions from Chapter 1.2 of the Lecture Course.

2. Classify also the following partial differential equations

$$u_{xx} + 4u_{xy} + 3u_{yy} + 3u_x - u_y + 2u = 0$$
$$a u_{xx} + 2a u_{xy} + a u_{yy} + b u_x + c u_y + u = 0.$$

Homework 2: *Laplace operator in polar coordinates*

(4 Points)

Show that the Laplace operator in polar coordinates has the following form:

$$\Delta u = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2}.$$

Homework 3: *Classification*

(2 Points)

Determine the regions of the (x, y) -plane, where the equation

$$(1 + x) u_{xx} + 2xy u_{xy} + y^2 u_{yy} + u_x = 0$$

is elliptic, hyperbolic or parabolic. Make a plot.