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Bergische Universität Wuppertal
Fachbereich C – Mathematik und Naturwissenschaften
Angewandte Mathematik / Numerische Analysis

Numerical Analysis and Simulation II: Partial Differential Equations (PDEs)

Exercise Sheet 7- Distributive Derivatives, Sobolev Spaces, Dual Spaces)

Return of Exercise Sheet: June 21, 2012 (before the lecture)

Homework 18: *Distributive derivatives* (3 Points)

1. Determine the *distributive derivative* of
$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0. \end{cases}$$
Does the *generalized derivative* of H exist?

2. Compute the first two distributive derivatives of
$$f(x) = |\sin x|.$$

Homework 19: (4 Points)

1. Let $\Omega \subset \mathbb{R}^n$ be bounded with $0 \in \Omega$. Prove that the function $u(x) = \|x\|_2^\sigma$ is an element of $H^1(\Omega)$, if $\sigma = 0$ or $2\sigma + n > 2$.

2. Let $\Omega = \{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 < r_0\}$ with $r_0 < 1$. Is the function
$$u(x, y) = \left(\log\left(\frac{1}{\sqrt{x_1^2 + x_2^2}}\right)\right)^k, \quad k < \frac{1}{2}$$
continuous? Does $u \in H^1(\Omega)$ hold?

Homework 20: (3 Points)

Let $(X, \|\cdot\|_X)$ be a normed space and X' the associated *dual space*, i.e. the space of continuous linear functionals on X. Show that

1. $\|F\|_{X'} := \sup\limits_{x \in X \setminus \{0\}} \dfrac{|F(x)|}{\|x\|_X}$ is a norm on X'.

2. $(X', \|\cdot\|_{X'})$ is a Banach space.

Lab-Exercise 3: *Scheme of higher order*

Solve the task from the Lab-Exercise 2 with the *scheme of higher order* from the lecture course, Chapter 3.7. Check again the computational order of convergence using the same kind of plot. Plot the solution for $N = 8$.