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Fachbereich C – Mathematik und Naturwissenschaften
Angewandte Mathematik / Numerische Analysis

Numerical Analysis and Simulation II: Partial Differential Equations (PDEs)

Exercise Sheet 9 - Weak formulation, Ansatz spaces

Return of Exercise Sheet: June 28, 2012 (before the lecture)

Homework 24: (2 Points)

In the lecture course we considered the boundary value problem

$$\begin{aligned} -u'' + \alpha u' + u &= f, \qquad x \in (0,1) \\ u'(0) = u'(1) &= 0 \end{aligned}$$

with ($\alpha = 1$) and derived a weak formulation with a non-symmetric, continuous und coercive bilinear form.

1. Let $\alpha = 1$. Argue why there exist exactly one weak solution $u \in H^1(0,1)$ if $f \in L^2(0,1)$.
2. Show: If $\alpha \in \mathbb{R}$ is sufficiently large, then the associated bilinear form is no longer coercive on $H^1(0,1)$.

Homework 25: (4 Points)

Consider the boundary value problem

$$-(a(x)u')' = 0, \quad x \in (-1,1), \quad u(-1) = 3,\ u(1) = 0$$

with

$$a(x) = \begin{cases} 1, & -1 \le x < 0, \\ 0.5, & 0 \le x \le 1. \end{cases}$$

State a *weak Formulation* of this problem and determine its solution.

Homework 26: *weak formulation* (4 Points)

Let Ω be a bounded domain with smooth boundary $\partial\Omega$, where $\partial\Omega = \Gamma_1 \cup \Gamma_2$, $\Gamma_1 \cap \Gamma_2 = \emptyset$.
Let c, f, g, p, q be continuous. Define $V := \{v \in H^1(\Omega) : v = 0 \text{ on } \Gamma_1\}$.
The function $u \in H^1(\Omega)$ with $u = g$ on Γ_1 fulfills the *weak formulation*

$$\int_\Omega \nabla u \nabla v + c\, uv\, dx + \int_{\Gamma_2} (pu - q)v\, ds = \int_\Omega f v\, dx \quad \text{für alle } v \in V.$$

Additionally, let $u \in C^2(\Omega) \cap C(\bar{\Omega})$.

State the associated classical problem and prove that u is a solution.

Lab-Exercise 4: *Linear Finite Elements*

Consider the boundary value problem

$$-(a(x)u')' = 1, \quad x \in (-1,1), \quad u(-1) = u(1) = 0$$

with

$$a(x) = \begin{cases} 1, & -1 \le x < 0, \\ 0.5, & 0 \le x \le 1. \end{cases}$$

1. Discretize the boundary value problem using *linear finite elements* on a uniform grid with the step size h.
2. State the resulting linear system.
3. Solve the linear system for the step sizes $h = 1/10$, $1/20$ and plot the solution.