Bergische Universität Wuppertal
Applied Mathematics and Numerical Analysis
Univ.-Prof. Dr. M. Ehrhardt

Winter Term 11/12

Exercise Sheet 10 to the Lecture Course "Computational Finance"
(Finite Elements and Divergence Free Formulation)

Task 1 (Calculating Options with Finite Elements) (5 Points)

Design an algorithm for the pricing of standard options by means of finite elements. To this end proceed as outlined in Section 5.3.

- Start with a simple version using an equidistant discretization step Δx.
- If this works properly change the algorithm to a version with nonequidistant x-grid. Distribute the nodes x_i closer around $x = 0$. Always place a node at the strike.

Task 2 (Divergence Free Formulation) (5 Points)

Prove the equivalence of

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 V_1}{\partial {S_1}^2} + r S_1 \frac{\partial V}{\partial S_1} - rV + \frac{1}{2}\sigma_2^2 S_2^2 \frac{\partial^2 V_1}{\partial {S_1}^2} + r S_2 \frac{\partial V}{\partial S_2} + \rho\sigma_1\sigma_2 S_1 S_2^2 \frac{\partial^2 V}{\partial S_1 \partial S_2} = 0 \quad (5.26)$$

and

$$-\nabla \cdot \big(D(x,y)\nabla u\big) + b(x,y)\nabla u + ru = \frac{\partial}{\partial \tau} u \quad (5.27a)$$

$$D(x,y) := \frac{1}{2}\begin{pmatrix} \sigma_1^2 x^2 & \rho\sigma_1\sigma_2 xy \\ \rho\sigma_1\sigma_2 xy & \sigma_2^2 y^2 \end{pmatrix},$$

$$b(x,y) := \frac{1}{2}\begin{pmatrix} (r - \sigma_1^2 - \rho\sigma_1\sigma_2/2)x \\ (r - \sigma_2^2 - \rho\sigma_1\sigma_2/2)y \end{pmatrix}, \quad (5.27b)$$

$$\nabla := \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix}.$$

- **Return** the solutions until Monday, January 23, **before** the lectures.
- **Return** the solutions of programming task until Monday, January 30, **before** the lectures.