Exercise Sheet 4 to the Lecture Course "Computational Finance"
(Generating Random Numbers)

<u>Task 1</u> (3 Points)

We consider the random number generator $N_i = 2N_{i-1} \mod 11$. For $(N_{i-1}, N_i) \in \{0, 1, \ldots, 10\}^2$ and integer tupels with $z_0 + 2z_1 = 0 \mod 11$ the equation

$$z_0 N_{i-1} + z_1 N_i = 0 \mod 11$$

defines families of parallel straight lines, on which all points (N_{i-1}, N_i) lie. These straight lines are to be analyzed. For which of the families of parallel straight lines are the gaps maximal?

<u>Task 2</u> (Lattice of the Linear Congruential Generator) (4 Points (2+2))

a) Show by induction over j

$$N_{i+j} - N_j = a^j (N_i - N_0) \mod M$$

b) Show for integer $z_0, z_1, \ldots, z_{m-1}$

$$\begin{pmatrix} N_i \\ N_{i+1} \\ \vdots \\ N_{i+m-1} \end{pmatrix} - \begin{pmatrix} N_0 \\ N_1 \\ \vdots \\ N_{m-1} \end{pmatrix} = (N_i - N_0) \begin{pmatrix} 1 \\ a \\ \vdots \\ a^{m-1} \end{pmatrix} + M \begin{pmatrix} z_0 \\ z_1 \\ \vdots \\ z_{m-1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \ldots & 0 \\ a & M & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a^{m-1} & 0 & \ldots & M \end{pmatrix} \begin{pmatrix} z_0 \\ z_1 \\ \vdots \\ z_{m-1} \end{pmatrix}$$

<u>Task 3</u> (Quality of Fibonacci-Generated Numbers) (3 Points)

Analyze and visualize the planes in the unit cube, on which all points fall that are generated by the Fibonacci recursion

$$U_{i+1} := (U_i + U_{i-1}) \mod 1.$$

- **Return** the solutions until Monday, November 28, **before** the lectures.