Bergische Universität Wuppertal
Applied Mathematics and Numerical Analysis
Univ.-Prof. Dr. M. Ehrhardt

Winter Term 11/12

Exercise Sheet 6 to the Lecture Course "Computational Finance"
(Finite Difference Methods)

Task 1 (Stability of the Fully Implicit Method) (5 Points)

The backward-difference method is defined via the solution of the equation

$$A := A_{\text{impl}} := \begin{pmatrix} 2\lambda+1 & -\lambda & & 0 \\ -\lambda & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ 0 & & \ddots & \ddots \end{pmatrix} \tag{1a}$$

where the vector $w(\nu)$ is implicitly defined as solution of the system of linear equations

$$Aw^{(\nu)} = w^{(\nu-1)} \quad \text{for} \quad \nu = 1, \dots, \nu_{\max} \tag{1b}$$

Prove the stability.
Hint: Consider $w^{(\nu)} = A^{-1}w^{(\nu-1)}$.

Task 2 (Crank-Nicolson Order) (5 Points)

Let the function $y(x,\tau)$ solve the equation

$$y_\tau = y_{xx}$$

and be sufficiently smooth. With the difference quotient

$$\delta_x^2 w_{i\nu} := \frac{w_{i-1,\nu} - 2w_{i\nu} + w_{i+1,\nu}}{\Delta x^2}$$

the local discretization error ϵ of the Crank-Nicolson method is defined

$$\epsilon := \frac{y_{i,\nu+1} - y_{i\nu}}{\Delta\tau} - \frac{1}{2}(\delta_x^2 y_{i\nu} + \delta_x^2 y_{i,\nu+1}).$$

Show

$$\epsilon = \mathrm{O}(\Delta\tau^2) + \mathrm{O}(\Delta x^2).$$

- **Return** the solutions until Monday, December 12, **before** the lectures.