Bergische Universität Wuppertal
Applied Mathematics and Numerical Analysis
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Winter Term 11/12

Exercise Sheet 7 to the Lecture Course "Computational Finance"
(Finite Difference Methods)

Task 1 (Semidiscretization) (2+3 +5 Points)

For a semidiscretization of the Black-Scholes equation

$$\boxed{\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0} \tag{BS}$$

consider the semidiscretized domain

$$0 \le t \le T, \qquad S = S_i := i\Delta S, \qquad \Delta S := \frac{S_{\max}}{m}, \quad i = 0, 1, \ldots, m$$

for some value $S_{\max}$. On this set of parallel lines define for $1 \le i \le m-1$ functions $w_i(t)$ as approximation to $V(S_i, t)$.

a) Using the standard second-order difference schemes, derive the system

$$\dot{w} + Bw = 0, \tag{ODE}$$

which up to boundary conditions approximates (BS).
Here w is the vector $(w_1, \ldots, w_{m-1})^\top$. Show that B is a tridiagonal matrix, and calculate its coefficients.

b) Use the BDF2 formula

$$f_i \approx \frac{4}{3}f_{i-1} - \frac{1}{3}f_{i-2} + \frac{2}{3}hf'(x_i) \tag{BDF2}$$

to show that

$$w^{(\nu)} = 4w^{(\nu-1)} - 3w^{(\nu-2)} + 2\Delta t B w^{(\nu-2)}$$

is a valid scheme to integrate (ODE).

c) Implement this scheme using MATLAB

- **Return** the solutions until Monday, December 19, **before** the lectures.
- **Return** the solutions of programming task until Monday, December 19, **before** the lectures.