Bergische Universität Wuppertal
Applied Mathematics and Numerical Analysis
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Winter Term 11/12

Exercise Sheet 9 to the Lecture Course "Computational Finance"
(B-Spline and Hat-Functions)

<u>Task 1</u> (Cubic B-Spline) (3+3+1 Points)

Suppose an equidistant partition of an interval be given with mesh-size $h = x_{k+1} - x_k$. Cubic B-splines have a support of four subintervals. In each subinterval the spline is a piece of polynomial of degree three. Apart from special boundary splines, the cubic B-splines φ_i are determined by the requirements

$$\begin{aligned} \varphi_i(x_i) &= 1 \\ \varphi_i(x) &\equiv 0 \quad \text{for} \quad x < x_{i-2} \\ \varphi_i(x) &\equiv 0 \quad \text{for} \quad x > x_{i+2} \\ \varphi &\in C^2(\mathbb{R}). \end{aligned}$$

To construct the φ_i proceed as follows:

a) Construct a spline $S(x)$ that satises the above requirements for the special nodes

$$\tilde{x}_k := 2 + k \quad \text{for} \quad k = 0, 1, \ldots, 4.$$

b) Find a transformation $T_i(x)$, such that $\varphi_i = S(T_i(x))$ satises the requirements for the original nodes.

c) For which i, j does $\varphi_i \varphi_j = 0$ hold?

<u>Task 2</u> (Finite-Element Matrices) (3 Points)

For the hat functions φ from Section 5.2 calculate for arbitrary subinterval $\mathcal{D}_k$ all nonzero integrals of the form

$$\int \varphi_i \varphi_j \, \mathrm{d}x, \qquad \int \varphi_i' \varphi_j \, \mathrm{d}x, \qquad \int \varphi_i' \varphi_j' \, \mathrm{d}x$$

and represent them as local 2×2 matrices.

- **Return** the solutions until Monday, January 16, **before** the lectures.