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Summer Term 2012

Bergische Universität Wuppertal
Fachbereich C – Mathematik und Naturwissenschaften
Angewandte Mathematik / Numerische Analysis

Numerical Analysis and Simulation II: Partial Differential Equations (PDEs)

Exercise Sheet 10 - Stiffnes matrix, Triangulation

Return of Exercise Sheet: July 5, 2012 (before the lecture)

Homework 27: (5 (1+1+1+1+1) Points)

Consider the boundary value problem

$$-\Delta u = \pi^2 \cos(\pi x), \qquad (x,y) \in \Omega = (0,1)^2$$
$$\frac{\partial u}{\partial n} = 0 \qquad \text{on} \quad \partial\Omega.$$

1. State the weak formulation.
2. The boundary value problem possess a unique solution if one additionally requires $\int_\Omega u = 0$. We choose as basis functions
$$b_1(x,y) = x - \frac{1}{2}, \qquad b_2(x,y) = \left(x - \frac{1}{2}\right)^3.$$
Show that the associated ansatz functions fullfill the above condition.
3. Determine the associated stiffnes matrix and the approximation of the right hand side.
4. Compute the approximate solution u_h.
5. Show, that $\frac{\partial u_h}{\partial n}(0,y) \neq 0$ holds.

Homework 28: (4 Points)

Consider the following Poisson equation on a unit square

$$-\Delta u = f, \quad \text{in } \Omega = (0,1)^2$$
$$u = 0 \quad \text{on } \partial\Omega.$$

The domain $\bar{\Omega}$ will be discretized uniformly with a grid of triangles with the mesh size h, as shown in Figure 1.

We choose the ansatz space

$$V_h := \{v \in C(\bar{\Omega}) : v \text{ is linear in each triangle and } v = 0 \text{ on } \partial\Omega\}.$$

1. What number is $N = \dim V_h$?
2. $v \in V_h$ is given globally by the values at the N grid points (x_j, y_j).
We choose a basis $\{\psi_i\}_{i=1}^N$ with $\psi_i(x_j, y_j) = \delta_{ij}$.
Determine the derivatives of the basis function ψ_Z in the triangles.

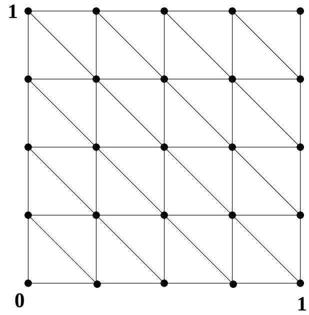


Abbildung 1:

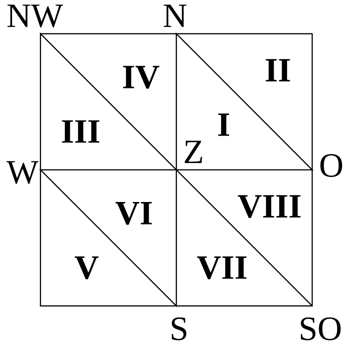
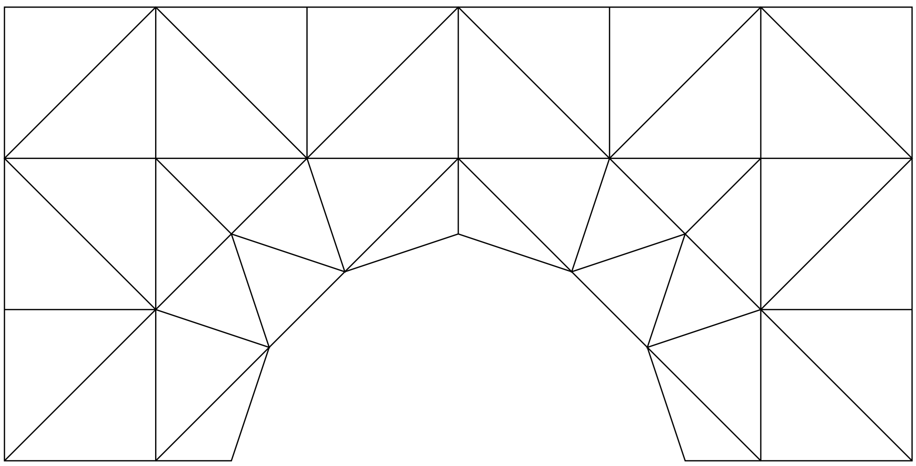


Figure 2:

3. Compute the matrix elements in the *stiffness matrix*.
4. What is the resulting linear system?

Homework 29: (1 Points)

Is the following *triangulation* feasible?



Homework 30: (2 Points)

Show the following formula for a *triangulation* of a simply connected domain

$$\#triangles + \#vertices - \#edges = 1.$$

Why is this no longer true for multiple connected domains?