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Bergische Universität Wuppertal
Fachbereich C – Mathematik und Naturwissenschaften
Angewandte Mathematik / Numerische Analysis

Numerical Analysis and Simulation II: Partial Differential Equations (PDEs)

Exercise Sheet 12 - The Method of simple Iteration, V-Cycle, W-Cycle, Remainder Projector

Return of Exercise Sheet: July 12, 2012 (before the lecture)

Homework 34: *Method of Simple Iteration* (4.5 (1+2+0.5+0.5+0.5) Points)

In the *Method of Simple Iteration* the parameter $\tau \in \mathbb{R}$ must be chosen such that

$$q(\tau) := \max_{\gamma_1 \le t \le \gamma_2} |1 - \tau t|\,, \qquad 0 < \gamma_1 \le \gamma_2$$

becomes minimal. Let $q(\tau^*) = \min\limits_{\tau \in \mathbb{R}} q(\tau)$. Sketch $|1 - \tau t|$ as a function of t for different values of τ and argue why

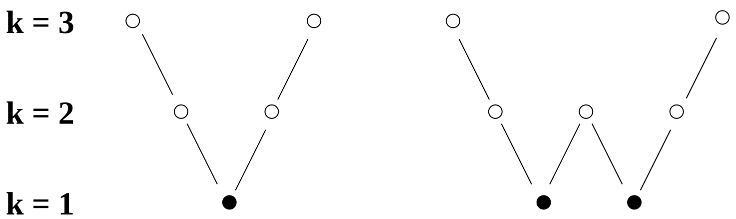
1. $q(\tau) < 1$ for $0 < \tau < \dfrac{2}{\gamma_2}$
2. $q(\tau^*) = \dfrac{\gamma_2 - \gamma_1}{\gamma_2 + \gamma_1} < 1$ for $\tau = \tau^* = \dfrac{2}{\gamma_1 + \gamma_2}$

holds. Explain using your draft the following remarks:

1. For $\tau = 2/\gamma_2$ one obtains the best possible, but still bad damping of low frequency components of the error (small eigen values).
2. For $\tau < 2/\gamma_2$ one can obtain a "good" damping of high frequency components of the error, e.g. on $[\tilde{\gamma}, \gamma_2]$ for $\tau = 2/(\tilde{\gamma} + \gamma_2)$.
3. $\tau = \tau^*$ is not suitable for the damping of high frequency error components!

Homework 35: (2.5 (1+1.5) Points)

The notations *V–cycle* and *W–cycle* stem from the fact that the shown schemes on 3 grids in the figure are similar to the letters V and W.



How do these schemes look on 4 grids?

Homework 36: (3 Points)

Prove Lemma 7.4 (Properties of the *Remainder projector*) from Chapter 7.4 of the lecture course.
Hint: Recall that for $v \in V_k$ holds: $\mathcal{P}_{k-1}v$ is the Galerkin–Approximation of v in V_{k-1} and use Theorem 6.6.1. Use the fact that for our model problem from Chapter 7.1 holds $\|v\|_{E,k,0} = \|v\|_{L^2(\Omega)}$, $v \in V_k$.

Lab-Exercise 1: *Multigrid MGLab*

Study the interactive MATLAB multigrid environment *MGLab*, see `http://www.cerfacs.fr/~douglas/mgnet/Codes/mglab/`. You will find a short description on the webpage of this lecture course.

Solve Lab-Exercise from Exercise Sheet 5 (Poisson Equation) with a two grid method without pre-smoothing, by modifying the files `demo2_run.m` and `get_rhs.m` appropriately.