Prof. Dr. M. Ehrhardt
D. Shcherbakov, M.Sc.
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Bergische Universität Wuppertal
Fachbereich C – Mathematik und Naturwissenschaften
Angewandte Mathematik / Numerische Analysis

Numerical Analysis and Simulation II: Partial Differential Equations (PDEs)

Exercise Sheet 3- ℓ^2-Stability following von Neumann

Return of Exercise Sheet: May 10, 2012 (before the lecture)

Homework 7: *Stability Analysis* (2 Points)

Perform the stability analysis of the θ–scheme of the lecture course using the *formal Fourier stability technique* of von Neumann.

Homework 8: ℓ^2*–Stabililität* (4 Points)

In order to solve the heat equation

$$u_t = u_{xx}, \qquad 0 < x < 1, \quad t > 0$$
$$u(0,t) = u(1,t) = 0$$

we introduce the discretization $D_t^0 u_j^n = D_x^2 u_j^n$ leading to the following *explicit 3-level scheme*

$$u_j^{(n+1)} = u_j^{(n-1)} + 2\gamma\,(u_{j+1}^{(n)} + u_{j-1}^{(n)}) - 4\gamma\, u_j^{(n)}, \tag{LF}$$

where $\gamma = k/h^2$ denotes the parabolic mesh ratio. Determine the order of convergence of this method. Is this scheme ℓ^2–stable (in the sense of von Neumann)?

Homework 9: ℓ^2*–Stability* (4 Points)

A modification of the method from Homework 2 for the solution of the heat equation leads to the *Dufort-Frankel scheme*

$$u_j^{(n+1)} = u_j^{(n-1)} + 2\gamma\,(u_{j+1}^{(n)} + u_{j-1}^{(n)} - u_j^{(n+1)} - u_j^{(n-1)}). \tag{DF}$$

Analyse the ℓ^2–stability properties and the order of consistency of this method.