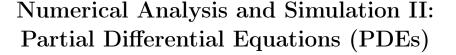
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Bergische Universität Wuppertal Fachbereich C – Mathematik und Naturwissenschaften Angewandte Mathematik / Numerische Analysis



Exercise Sheet 7- Distributive Derivatives, Sobolev Spaces, Dual Spaces)

Return of Exercise Sheet: June 21, 2012 (before the lecture)

Homework 18: Distributive derivatives

1. Determine the *distributive derivative* of

$$H(x) = \begin{cases} 1 & x \ge 0\\ 0 & x < 0. \end{cases}$$

Does the generalized derivative of H exist?

2. Compute the first two distributive derivatives of

$$f(x) = |\sin x|.$$

Homework 19:

- 1. Let $\Omega \subset \mathbb{R}^n$ be bounded with $0 \in \Omega$. Prove that the function $u(x) = \|x\|_2^{\sigma}$ is an element of $H^1(\Omega)$, if $\sigma = 0$ or $2\sigma + n > 2$.
- 2. Let $\Omega = \{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 < r_0\}$ with $r_0 < 1$. Is the function

$$u(x,y) = \left(\log\left(\frac{1}{\sqrt{x_1^2 + x_2^2}}\right)\right)^k, \quad k < \frac{1}{2}$$

continuous? Does $u \in H^1(\Omega)$ hold?

Homework 20:

Let $(X, \|\cdot\|_X)$ be a normed space and X' the associated *dual space*, i.e. the space of continuous linear functionals on X. Show that

1. $||F||_{X'} := \sup_{x \in X \setminus \{0\}} \frac{|F(x)|}{||x||_X}$ is a norm on X'.

2. $(X', \|\cdot\|_{X'})$ is a Banach space.

Lab-Exercise 3: Scheme of higher order

Solve the task from the Lab-Exercise 2 with the scheme of higher order from the lecture course, Chapter 3.7. Check again the computational order of convergence using the same kind of plot. Plot the solution for N = 8.

(3 Points)

(4 Points)

(3 Points)