

# Mathematical Modelling and Numerical Algorithms for Simulation of Oil Pollution

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## Abstract

This talk deals with the mathematical modeling and algorithms for the problem of oil pollution. For solving this task we derive the adjoint problem for the advection-diffusion equation describing the propagation of oil slick after an accident, which we call the main problem.

We prove a fundamental equality between the solutions of the main and the adjoint problems. Based on this equality we propose a novel method for the identification of the pollution source location and the accident time of oil emission.

This approach is illustrated on an example for an accident in the offshore of the central part of the Vietnamese coast. Numerical simulations demonstrate the effectiveness of the proposed method. Besides, the method is verified for 1D model of substance propagation.

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