

# Absorbing Boundary Conditions for Hyperbolic Systems

M. EHRHARDT

Lehrstuhl für Angewandte Mathematik und Numerische Analysis, Fachbereich C – Mathematik und Naturwissenschaften, Bergische Universität Wuppertal, Gaußstr. 20, 42119 Wuppertal, Germany, ehrhardt@math.uni-wuppertal.de

## Abstract

This talk deals with absorbing boundary conditions (ABCs) for hyperbolic systems in one and two space dimensions. We prove the strict well-posedness of the resulting initial boundary value problem in 1D. Afterwards we establish the GKS-stability of the corresponding Lax-Wendroff-type finite difference scheme. Hereby, we have to extend the classical proofs, since the (discretized) ABCs do not fit the standard form of boundary conditions for hyperbolic systems.

In the second part we present the approach of deriving so-called discrete absorbing boundary conditions, i.e. ABCs constructed on a purely discrete level. These discrete ABCs are better adapted to the interior scheme: they lead to less unphysical reflections and the resulting overall scheme has better stability properties.

Finally, we sketch briefly how ABCs can be derived for nonlinear hyperbolic systems.

## References

- [1] T. Amro, A. Arnold and M. Ehrhardt, *Optimized Far Field Boundary Conditions for Hyperbolic Systems*, Preprint 12/02, University of Wuppertal, January 2012.
- [2] M. Ehrhardt, *Discrete Artificial Boundary Conditions*, Ph.D. Thesis, TU Berlin, 2001.
- [3] M. Ehrhardt, *Absorbing Boundary Condition for Hyperbolic Systems*, Numer. Math: Theory, Meth. Appl. **3** (2010), 295–337.
- [4] B. Engquist and A. Majda, *Radiation Boundary Conditions for Acoustic and Elastic Wave Calculations*, Comm. Pure Appl. Math. **32** (1979), 313–357.
- [5] M. Grote and C. Kirsch, *Far-field evaluation with nonreflecting boundary conditions*, Proceedings of the Ninth Internat. Conf. on Hyperbolic Problems (HYP2002), Pasadena, USA, 2002.
- [6] B. Gustafsson, H.-O. Kreiss and A. Sundström, *Stability Theory for Difference Approximations of Mixed Initial Boundary Value Problems. II*, Math. Comp. **26** (1972), 649–686.
- [7] T. Hagstrom, D. Givoli and T. Warburton, *Radiation boundary conditions for time-dependent waves based on complete plane wave expansions*, J. Comput. Appl. Math. **234** (2010), 1988–1995.
- [8] L. Halpern, *Absorbing Boundary Conditions for the Discretization Schemes for the One-Dimensional Wave Equation*, Math. Comp. **38** (1982), 415–429.
- [9] G.W. Hedstrom, *Nonreflecting boundary conditions for nonlinear hyperbolic systems*, J. Comput. Phys. **30** (1979), 222–237.
- [10] K. Hoke, *Discrete transparent boundary conditions for hyperbolic systems*, Diploma Thesis, Institute for Mathematics, TU Berlin, 2006. (in german)
- [11] D. Kröner, *Absorbing Boundary Conditions for the Linearized Euler Equations in 2-D*, Math. Comput. **57** (1991), 153–167.
- [12] G. Lill, *Diskrete Randbedingungen an künstlichen Rändern*, Ph.D. Thesis, TU Darmstadt, 1992.

- [13] C.W. Rowley and T. Colonius, *Discretely Nonreflecting Boundary Conditions for Linear Hyperbolic Systems*, J. Comput. Phys. **157** (2000), 500–538.
- [14] K.W. THOMPSON, *Time dependent boundary conditions for hyperbolic systems*, J. Comput. Phys. **68** (1987), 1–24.
- [15] K.W. Thompson, *Time-dependent boundary conditions for hyperbolic systems. II*, J. Comput. Phys. **89** (1990), 439–461.
- [16] L. Wagatha, *Approximation of Pseudodifferential Operators in Absorbing Boundary Conditions for Hyperbolic Equations*, Numer. Math. **42** (1983), 51–64.
- [17] J.C. Wilson, *Derivation of boundary conditions for the artificial boundaries associated with the solution of certain time dependent problems by Lax–Wendroff type difference schemes*, Proc. Edinb. Math. Soc., II. Ser. **25** (1982), 1–18.