A Review of Transparent and Artificial Boundary Conditions Techniques for Linear and Nonlinear Schrödinger Equations

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Abstract In this review we discuss different techniques to solve numerically the time-dependent Schrödinger equation on unbounded domains. We present and compare several approaches to implement the classical transparent boundary condition into finite difference and finite element discretizations. We present in detail the approaches of the authors and describe briefly alternative ideas pointing out the relations between these works. We conclude with several numerical examples from different application areas to compare the presented techniques. Here we mainly focus on the one-dimensional problem.

References

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